Section 2.3 The Chain Rule (Minimum problems: 21 – 37 odds 41, 47, 49, 51)

#1-10: Find $f[g(x)]$, and do not simplify your answer!!!

1) $f(x) = x^3$; $g(x) = x^2 + 1$

replace the "x" in the f-function with the right side of the g-function.

$f[g(x)] = (x^2 + 1)^3$

3) $f(x) = 5x^2$; $g(x) = 3x - 4$

replace the "x" in the f-function with the right side of the g-function.

$f[g(x)] = 5(3x - 4)^2$

5) $f(x) = 7x^{2/3}$; $g(x) = 5x + 4$

replace the "x" in the f-function with the right side of the g-function.

$f[g(x)] = 7(5x + 4)^{2/3}$

7) $f(x) = e^x$; $g(x) = x^2 + 2x + 1$

replace the "x" in the f-function with the right side of the g-function.

$f[g(x)] = e^{x^2+2x+1}$

9) $f(x) = ln(x)$; $g(x) = 3x + 5$

replace the "x" in the f-function with the right side of the g-function.

$f[g(x)] = \ln{(3x + 5)}$

#11-16: Create two functions $f(x)\ and\ g(x)$ whose composition is the given function $f[g(x)]$

$g - function$ the inside of the parenthesis, or expression under the radical.

$f - funtion$ the problem with the inside of the parenthesis or the expression under the radical changed to "x"

11) $f[g(x)] = (7x - 3)^2$

$f(x) = x^2$ $g(x) = 7x - 3$

$g - function$ the inside of the parenthesis, or expression under the radical.

$f - funtion$ the problem with the inside of the parenthesis or the expression under the radical changed to "x"

13) $f[g(x)] = 2(4x+7)^5$

$f(x) = 2x^5$ $g(x) = 4x + 7$

$g - function$ the inside of the parenthesis, or expression under the radical.

$f - funtion$ the problem with the inside of the parenthesis or the expression under the radical changed to "x"

15) $f[g(x)] = \sqrt{x+5}$

$f(x) = \sqrt{x}$ $g(x) = x + 5$

#17-24: Use the Chain rule to find the derivative of each function.

$\frac{d}{dx} f[g(x)] = g'(x) * f'[g(x)]$

CHAIN RULE short cut to find a derivative of a problem written in the form:

$$f(x) = a[g(x)]^n$$

$$f'(x) = n * a * g'(x)[g(x)]^{n-1}$$

$$f(x) = a(inside\ parenthesis)^n$$

$$f'(x) = n * a * (derivative\ of\ inside\ of\ parenthesis)^{n-1}$$

17) $h(x) = (7x-3)^2$

$a=1 \quad n=2$

$g(x)=7x-3$

$g'(x)=7$

$h'(x) = 2 \cdot 1 \cdot 7(7x-3)^1$

$h'(x) = 14(7x-3)$

19) $h(x) = 2(4x+7)^5$

$a=2 \quad n=5$

$g(x)=4x+7$

$g'(x)=4$

$h'(x) = 5 \cdot 2 \cdot 4(4x+7)^4$

$h'(x) = 40(4x+7)^4$

CHAIN RULE short cut to find a derivative of a problem written in the form:

$$f(x) = a[g(x)]^n$$

$$f'(x) = n * a * g'(x)[g(x)]^{n-1}$$

$$f(x) = a(inside\ parenthesis)^n$$

$$f'(x) = n * a * (derivative\ of\ inside\ of\ parenthesis)^{n-1}$$

21) $h(x) = 4(2x-1)^3$

$n = 3 \quad a = 4$

$g(x) = 2x-1$

$g'(x) = 2$

$h'(x) = 3 \cdot 4 \cdot 2(2x-1)^2$

$h'(x) = 24(2x-1)^2$

23) $h(x) = (x^2+6x+1)^3$

$n = 3 \quad a = 1$

$g(x) = x^2+6x+1$

$g'(x) = 2x+6$

$h'(x) = 3 \cdot 1(\underbrace{2x+6}_{GCF\ 2})(x^2+6x+1)^2$

$2 \cdot 3 \cdot 1$

$h'(x) = 6(x+3)(x^2+6x+1)^2$

#25-46: Find the derivative of each function.

CHAIN RULE short cut to find a derivative of a problem written in the form: $$f(x) = a[g(x)]^n$$ $$f'(x) = n * a * g'(x)[g(x)]^{n-1}$$ $$f(x) = a(inside\ parenthesis)^n$$ $$f'(x) = n * a * (derivative\ of\ inside\ of\ parenthesis)^{n-1}$$

25) $y = \ 5x(2x - 4)^3$

First factor $5x$	Second Factor $(2x - 4)^3$
Derivative 5	Derivative $3 * 2(2x - 4)^2$ $6(2x - 4)^2$
$5x * 6(2x - 4)^2$ $30x(2x - 4)^2$	Cross multiply $5(2x - 4)^3$

$y' = 30x(2x-4)^2 + 5(2x-4)^3$

GCF 5

$y' = 5(6x(2x-4)^2 + (2x-4)^3)$

$y' = 5(2x-4)^2(6x + 2x - 4)$

$y' = 5(2x-4)^2(8x-4)$

$y' = 5 \cdot 2 \cdot 2 \cdot 4(x-2)^2(2x-1)$

$y' = 80(x-2)^2(2x-1)$

27) $g(t) = 6t^2(2t+5)^2$

First factor $6t^2$	Second Factor $(2t+5)^2$
Derivative $12t$	Derivative $2 * 2(2t+5)^1$ $4(2t+5)$
$6t^2 * 4(2t+5)$ $24t^2(2t+5)$	Cross multiply $12t(2t+5)^2$

$$g'(t) = 24t^2(2t+5) + 12t(2t+5)^2$$

$$g'(T) = 12T\left(2T(2t+5) + (2T+5)^2\right)$$

$$g'(T) = 12T(2T+5)(2T+2T+5)$$

$$g'(T) = 12(2T+5)(4T+5)$$

29) $h(y) = (6y - 3)(5y + 4)^2$

First factor $6y - 3$	Second Factor $(5y + 4)^2$
Derivative 6	Derivative $2 * 5(5y + 4)^1$ $10(5y + 4)$
$(6y - 3)10(5y + 4)$ $10(6y - 3)(5y + 4)$	Cross multiply $6(5y + 4)^2$

$$h'(y) = 10(\underbrace{6y-3}_{3})(5y+4) + 6(5y+4)^2$$

$10 \cdot 3$

$$h'(y) = \underbrace{30}_{6}(2y-1)(5y+4) + \underbrace{6}_{6}(5y+4)^2$$

$$h'(y) = 6(5(2y-1)(5y+4) + (5y+4)^2)$$

$$h'(y) = 6(5y+4)(5(2y-1) + 5y+4$$

$$10y - 5 + 5y + 4$$

$$h'(y) = 6(5y+4)(15y-1)$$

31) $y = \frac{2}{(3x-4)^2}$

Denominator $(3x-4)^2$	Numerator 2
Derivative $2 * 3(3x-4)$ $6(3x-4)$	Derivative 0
Cross multiply $(3x-4)^2 * 0$ 0	$2 * 6(3x-4)$ $12(3x-4)$

$$y' = \frac{0 - 12(3x-4)}{((3x-4)^2)^2} = (3x-4)^4$$

$$y' = \frac{-12(3x-4)}{(3x-4)(3x-4)(3x-4)(3x-4)}$$

$$y' = \frac{-12}{(3x-4)^3}$$

33) $y = \frac{2x}{(3x-4)^4}$

Denominator $(3x-4)^4$	Numerator $2x$
Derivative $4 * 3(3x-4)^3$ $12(3x-4)^3$	Derivative 2
Cross multiply $(3x-4)^4 * 2$ $2(3x-4)^4$	$12(3x-4)^3 * 2x$ $24x(3x-4)^3$

$$y' = \frac{2(3x-4)^4 - 24x(3x-4)^3}{\left((3x-4)^4\right)^2}$$

$$y' = \frac{2(3x-4)^3(3x-4-12x)}{(3x-4)^8}$$

$$y' = \frac{2\cancel{(3x-4)}\cancel{(3x-4)}\cancel{(3x-4)}(-9x-4)}{\cancel{(3x-4)}\cancel{(3x-4)}\cancel{(3x-4)}(3x-4)^5}$$

$$y' = \frac{2(-9x-4)}{(3x-4)^5} = \frac{-2(9x+4)}{(3x-4)^5}$$

#35-40:

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

35) $f(x) = (2x-3)^2$

a)

$f'(x) = 2 * 2(2x-3)$

$f'(x) = 4(2x-3)$

$4(2x-3) = 0$

$4 = 0$ No Solution

$2x-3=0$

$2x=3$

$x = 3/2$

$\boxed{\text{Part a answer } x = 3/2}$

b) horizontal tangent $\rightarrow m=0$

y-coord point

$y = f(3/2) = (2 \cdot \frac{3}{2} - 3)^2 = 0$

Point $(3/2, 0)$ $m=0$

$y-0 = 0(x-3/2)$

$y-0=0$

Answer PART b $y=0$

#35-38:

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

37) $y = 5(x + 3)^4$

a)

$f'(x) = 20 * 1(x+3)^3$

$f'(x) = 20(x+3)^3$

$20(x+3)(x+3)(x+3) = 0$

$20 = 0$

No Solution

$x+3=0$

$x=-3$

$\boxed{\text{Answer part a } x=-3}$

b) $m=0$ (horizontal line)

$y = f(-3) = 5(-3+3)^4 = 0$

point $(-3, 0)$ $m=0$

$y-0 = 0(x-(-3))$

$y-0=0$

Answer part b

$\boxed{y=0}$